Kinetic equation with exact charge conservation*

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We investigate the chemical equilibration of rare particles which are subject to a conservation law, such as kaons in low energy heavy ion collisions. We formulate the kinetic master equation describing production of charged particles which are created or destroyed only in pairs due to conservation of their Abelian charge required by an $U(1)$ internal symmetry. The crucial step is to preserve the two particle correlation induced by the $U(1)$ symmetry. To this end one starts from the general rate equation for the process $a_1 + a_2 \leftrightarrow b_1 + b_2$, where b_1 and b_2 are subject to the $U(1)$ charge conservation:

$$\frac{d\langle N_{b_1}\rangle}{d\tau} = \frac{G}{V}\langle N_{a_1}\rangle\langle N_{a_2}\rangle - \frac{L}{V}\sum_{i,j} ijP_{i,j}. \qquad (1)$$

Here G and L stand for the gain and loss term and $P_{i,j}$ is the probability to find i number of particle b_1 *and* j number of particle b_2 in an event.

In case that particles b_1 and b_2 carry opposite units of a charge corresponding to an $U(1)$ internal symmetry we have

$$\begin{aligned} P_{i,j} &= P_i\ \delta_{ij}, \\ \sum_{i,j} ijP_{i,j} &= \sum_i i^2 P_i \equiv \langle N^2\rangle \end{aligned} \qquad (2)$$

The grand canonical approximation corresponds to $\langle N^2\rangle \simeq \langle N\rangle^2$, which is true in case of many particles but fails if rare particles are considered.

The general master equation for the time evolution of the probability P_n to find n particles is then given by:

$$\begin{aligned} \frac{dP_n}{d\tau} &= \frac{G}{V}\langle N_{a_1}\rangle\langle N_{a_2}\rangle\ [P_{n-1} - P_n] \\ &- \frac{L}{V}\left[n^2 P_n - (n+1)^2 P_{n+1}\right], \end{aligned} \qquad (3)$$

It can be easily shown that the equilibrium solution ($t \to \infty$) for the average number of particles agrees with the results from the canonical distribution [1]:

$$\langle N\rangle_{\rm eq} = \sqrt{\epsilon}\,\frac{I_1(2\sqrt{\epsilon})}{I_0(2\sqrt{\epsilon})}, \qquad (4)$$

with $\epsilon \equiv G\langle N_{a_1}\rangle\langle N_{a_2}\rangle/L$.

Furthermore, we find that if the number of particles subject to the $U(1)$ charge conservation is small, $\langle N\rangle \ll 1$, the relaxation time is much shorter in the canonical system as compared to the grand canonical approximation. This finding has important implication on the method of used in transport simulations for kaon production at low energies.

[1] J. Cleymans and K. Redlich, Phys. Rev. **C60**, 054908 (1999)

* LBNL-46896

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